ICM

MARCH 2017

NUMBER & LOGIC

Instructions to candidates:

a) Time allowed: Three hours (plus an extra ten minutes' reading time at the start – do not write anything during this time)
b) Answer any FIVE questions
c) All questions carry equal marks. Marks for parts of questions are shown in [ ]. The amount of time spent on each question and the length of your answer should be proportional to the mark allocation
d) In numerical questions, candidates must show and explain the method of working to obtain full marks
e) Non-programmable calculators are permitted in this examination. Where calculators are used ensure that you write down all intermediate values obtained from a calculator. Always explain in words what you are calculating
f) Ensure that you leave numeric answers in the format required by the question
g) In binary questions, you are advised to group your binary digits into recognisable units of, say, 3 digits. SPACE OUT BINARY CALCULATIONS SO THAT COLUMNS ARE CLEARLY ALIGNED
h) Ensure that you pay particular attention to words underlined, in CAPITALS or in **bold**. FEW OR NO MARKS will be awarded to any question where these are ignored
i) No computer equipment, books or notes may be used in this examination
j) GRAPH PAPER may be issued as required

1. Matrices A and B are defined as:

$$A = \begin{vmatrix} -14 & 6 & 4 \\ 10 & -4 & -2 \\ 6 & -2 & -2 \end{vmatrix} \qquad B = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & -1 \end{vmatrix}$$

a) Perform the matrix calculation A . B. [6]
b) Write down the equations below in matrix format.
$x + y + z = 6$
$2x + y + 3z = 11$
$x + 2y - z = 5$ [3]
c) Explain WHY the result obtained in a) can be used to solve the equation in b). [2]
d) Using this result, solve the simultaneous equations and state the values of x, y and z. [9]

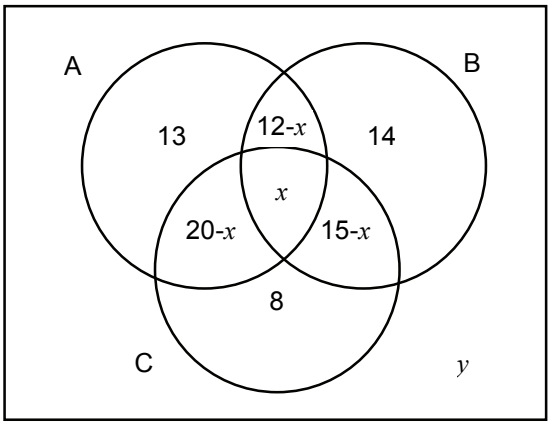
2. X, Y and Z represent binary numbers as follows:
X = 101011
Y = 11110010
Z = 11010111

Showing all working, carry out the following calculations **entirely in binary**:
a) X + Y + Z [3]
b) Y * X [4]
c) Z / X [8]
d) Y-Z [5]

continued overleaf

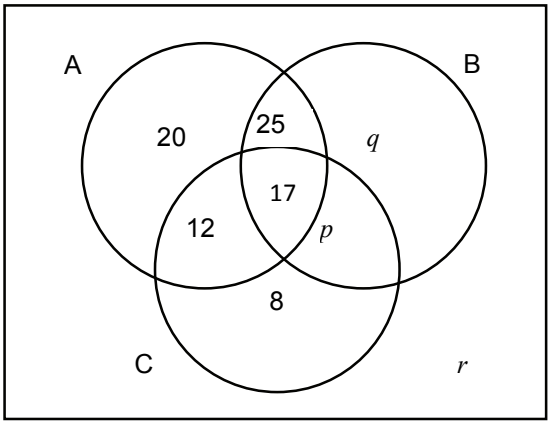
3. a) In a study to discover how people watch movies these days 100 people were surveyed.
The options were – TV (A), Computer (B) or Mobile device (C)
The Venn diagram below represents the results.
26 of those people surveyed said they didn't watch movies.

i Showing your working – calculate how many people used all 3 devices to watch movies (x). [5]



ii Find the value of n((A ∪ B)' ∩ C) [2]

b) A cafe asked 100 customers whether they drink any of 3 coffee drinks (Americano, Breve and Cappuccino). The results are shown in the Venn diagram below.



42 said they drink Cappuccino
40 said they drink only one kind of coffee

i Find the values of the following: [3]
- Customers who drink Cappuccino and Breve
- Customers who drink only Breve
- Customers who don't drink coffee

ii Find: [3]
- n(A')
- n((A ∪ B) ∩ C')

iii Find the probability that a customer likes Americano but not Cappuccino. [1]

iv Two customers are chosen at random from those who like Breve. Find the probability that they each like exactly one other type of coffee. [3]

c) Use a Venn diagram to illustrate the following: [3]
(A ∪ B)' = A' ∩ B'

4. Show ALL working in this question.

a) In an arithmetic series, the sum of the first five terms is 30. Also, the sum of the first two terms is equal to the third term. Determine the first term, the common difference and the value of the tenth term. [10]

b) A is an arithmetic progression. G is a geometric progression. Both the first two terms of these two series are 6 and 12. Find the sum of the first TEN terms of each series and the difference between the two sums. [10]

continued overleaf

5. a) Show how 16-bit location can be used to store EACH of the following AND give a clear numeric example for EACH showing the FULL bit patterns:
   i BCD
   ii ASCII
   iii A floating point number
   iv A mid-point number [3 each]
   b) For EACH, explain how a computer program would be able to detect that the particular format was being used. [4]
   c) The number in a single byte has the binary value 10 110 101.
   Two's complement arithmetic is being used.
   i Explain why it is clear this number is negative.
   ii Write down the bit values of EACH bit in a byte which uses two's complement format. [4]

6. a) X is a 16-bit location.
   i Show how the decimal value 593 would be held in binary-coded decimal format. [3]
   ii Explain why the binary value 1011 0000 1111 0001 cannot be interpreted as BCD. [2]
   b) Given that the ASCII codes for A to Z are 65 to 90 (decimal) and for a to z they are 97 to 122, write an algorithm to convert all lower case letters in a string (S) of 20 characters into upper case. Some of the characters in the string may not be alphabetic and should be left untouched. [10]
   c) Show, with the aid of a diagram, how a low-level instruction is held in a memory location. You can choose the size of the location. [5]

7. A, B and C are three FLOATING-POINT numbers held in NORMALISED form, each in a 2-byte register. The EXPONENT part is held in the 6 least significant bits and the MANTISSA takes up the remaining digits.
   a) Define the FOUR terms underlined above. [4]
   b) The binary values of A and B are:
   A = 0101 1100 0000 0001 B = 0111 0110 0000 0101
   i Calculate the decimal value of EACH exponent and mantissa. [6]
   ii Calculate the precise value held in each of A and B, giving each answer as an integer and a fraction where the fraction should be in its lowest form and with format x / y. [4]
   iii A and B are added together and the sum placed in C. Show all the stages, in binary, of adding these normalised floating point numbers and the final sum normalised in C. [6]

8. A manufacturing company makes two machines A and B. Details of the manufacture of each machine are shown in this table.

Machine	Time to make (man-days)	Cost to make (£)
A	100	20,000
B	300	30,000

At a given time, the company has £240,000 and 1,800 man-days at its disposal.
   a) Write down inequalities for these if the company makes **a** machines of type A and **b** of type B. [4]
   b) Draw a graph plotting these inequalities. Plot **a** vertically and **b** horizontally. Ensure the graph is as large as possible on the given paper. LIGHTLY shade the INVALID sides of lines, being careful not to obscure the lines themselves. [6]
   c) The profit from making one A machine is £2,500 and for B is £5,000. Write down a profit equation and then plot ANY profit line on the graph – mark it P. Slide this line so that it shows MAXIMUM profit.
   For MAXIMUM profit, write down the number of each machine being made and the resulting profit. [5]
   d) If the manufacturer then decides that he must make at least as many B machines as A machines, determine from your graph the maximum profit now possible. [5]