ICM

MARCH 2016

NUMBER & LOGIC

Instructions to candidates:

a) Time allowed: Three hours (plus an extra ten minutes' reading time at the start – do not write anything during this time)
b) Answer any FIVE questions
c) All questions carry equal marks. Marks for parts of questions are shown in [ ]
d) In numerical questions, candidates must show and explain the method of working to obtain full marks
e) Non-programmable calculators are permitted in this examination. Calculators should be used in all the non-binary questions. However, ensure that you write down all intermediate values obtained from a calculator. Always explain in words what you are calculating
f) Ensure that you leave numeric answers in the format required by the question
g) Ensure that you pay particular attention to words underlined, in CAPITALS or in **bold**. FEW OR NO MARKS will be awarded to any question where these are ignored
h) No computer equipment, books or notes may be used in this examination

1. X, Y and Z are binary integers as follows:
X = 101101 Y = 101 Z = 10101
Showing all stages of working, perform the following calculations IN BINARY. You are advised to space out numbers so that they are in strict columns.
   a) Find X + Y + Z. [3]
   b) Find X – Z. [2]
   c) Find Y × Z. [4]
   d) Find X / Z. Leave your answer accurate to 3 binary places. [4]
   e) Convert X and Z to decimal and show that your answers to b) and d) are correct. [4]
   f) IF the decimal number 24.68 is to be converted to binary as accurately as possible, how many BINARY places would be needed? Explain your answer. [3]

2. a) Consider the decimal values of the contents of the three memory locations P, Q and R.
P = 0110 0000 0011 1000
Q = 1011 0000 1100 1000
R = 0110 1010 0000 1010
      i Calculate the decimal value of the contents of P if it holds a midpoint fractional value with the binary point assumed to be in the middle. Give the fractional part as a fraction in its lowest cancelled form. [4]
      ii Calculate the decimal value of the contents of Q if the value is held in two's complement binary form. [4]
      iii Calculate the decimal value of the contents of R if the value is held in floating point form with the left 10 bits used for the mantissa and the right 6 bits for the exponent. [4]
   b) The table shows the contents of memory of a computer (given in hexadecimal addresses) at a particular instance in time.

	START ADDRESS	END ADDRESS
Program A	0	39FF
Program B	?	?
Program C	7700	ACFF

      i Calculate the start and end addresses for program B if the three programs are held one after another without any gaps. [2]
      ii Calculate the size of program B in both hexadecimal AND decimal. [4]
      iii A second copy of program B is loaded immediately after program C. Determine the hexadecimal address of the end of that program. [2]

continued overleaf

3. a) Write down an algorithm (for example pseudocode or flowchart) for converting **any** decimal number to binary. [8]

b) Showing relevant working, convert the following:

i Hex value 17FB into decimal [4]

ii Decimal 4321 into hex [4]

iii Binary string 00110010 into decimal [2]

iv Hex value E9 into binary [2]

4. a) The equation $x^4 + x^2 - 80 = 0$ has three solutions.

i Using a calculator and the iterative formula given below, determine the solution near to x = 3 to five decimal places. You should show all working and all intermediate values towards the final answer.

$x1 = x0 - (x0^4 + x0^2 - 80) / (4x0^3 + 2x0)$

ii Explain how you know that this answer is correct to five decimal places. [15]

b) Explain why iterative methods to solve equations are beneficial to the mathematician/scientist. [5]

5. a) State a rule for multiplying matrices. [2]

b) State a rule for addition or subtraction of matrices. [2]

c) Add the matrices A and B [2]

$$A = \begin{vmatrix} 203 & 10 \\ -12 & 20 \end{vmatrix} \qquad B = \begin{vmatrix} 4 & 2 \\ -3 & 3 \end{vmatrix}$$

d) Subtract the matrix N **from** matrix M (i.e. M – N) [2]

$$M = \begin{vmatrix} 22 & 18 \\ -12 & 2 \end{vmatrix} \qquad N = \begin{vmatrix} 14 & 26 \\ -8 & 5 \end{vmatrix}$$

e) Perform matrix multiplication P × Q [6]

$$P = \begin{vmatrix} 2 & 0 & 1 \\ -2 & 0 & -1 \end{vmatrix} \qquad Q = \begin{vmatrix} 4 & 2 \\ -3 & 3 \\ 2 & 4 \end{vmatrix}$$

f) Multiply the matrices X and Y below [6]

$$X = \begin{vmatrix} 4 & 8 \\ 0 & 2 \\ 1 & 6 \end{vmatrix} \qquad Y = \begin{vmatrix} 5 & 2 \\ 9 & 4 \end{vmatrix}$$

6. a) An item incurs a **loss** of 5% by selling for £114. At what price should the item be sold to earn 5% **profit**? Show your working. [6]

b) A company uses large quantities of "widgets" and buys a consignment several times each year. Every year 30,000 widgets are used. One widget costs £20 and, in addition, there is a handling charge of £40 for each order. A handling/carrying cost of 12% per item is also applied. The company wishes to order as economically as possible.

i Explain what is meant by EOQ (Economic Order Quantity), stating what factors are taken into account. [2]

ii Write down the formula for calculating EOQ and calculate the EOQ for this product. [6]

c) £1,000 is invested for 3 years at an investment rate of 5%.

i Calculate the simple interest on this amount. [2]

ii Calculate the compound interest on the same investment. [4]

continued overleaf

7. a) The following table shows the grouped data of sales for a particular product across 50 stores:

Number of Stores	2	5	25	10	8
Total Sales	120 to 130	130 to 140	140 to 150	150 to 160	160 to 170

Calculate the following and in EACH case show all working and reasoning:

i The arithmetic mean [4]

ii The standard deviation [6]

iii Explain what the standard deviation represents [2]

b) The frequency table below shows the results of 8 questions in a school quiz:

Question	1	2	3	4	5	6	7	8
Number of Correct Answers	4	6	7	14	10	8	7	4

Showing ALL working, determine:

i the mean number of correct answers achieved [1]

ii the range of number of correct answers [1]

iii the median number of correct answers [3]

c) Explain what is meant by the **modal value** of a set of data – give an example to illustrate your explanation. [3]

8. a) 200 holidaymakers were asked whether they had visited any of these three countries – Austria, Belgium and Croatia.

- 30 people have not visited any of these countries
- 10 people have been to all three countries
- 25 people have been to Belgium and Austria
- 20 people have been to Croatia and Austria but not to Belgium
- 65 people have been to exactly two of the countries
- 165 people have been to Croatia or to Belgium
- 120 people have been to Belgium or to Austria

i Draw a Venn diagram to represent this data. [4]

ii Use your diagram to calculate: [6]

- the number of people who had only visited Austria
- the number of people who had only visited Belgium
- the number of people who had only visited Croatia

b) If a coin is flipped 3 times, what is the probability of getting two tails and one head? Show your working. [6]

c) Two dice are rolled and the results are added together.

i Find the probability that their sum is an odd number. [2]

ii Find the probability that their sum is greater than 6. [2]