ICM

SEPTEMBER 2015

NUMBER & LOGIC

Instructions to candidates:

a) Time allowed: Three hours (plus an extra ten minutes' reading time at the start – do not write anything during this time)
b) Answer any FIVE questions
c) All questions carry equal marks. Marks for parts of questions are shown in [ ]. The amount of time spent on each question and the length of your answer should be proportional to the mark allocation
d) In numerical questions, candidates must show and explain the method of working to obtain full marks
e) Non-programmable calculators are permitted in this examination. Where calculators are used ensure that you write down all intermediate values obtained from a calculator. Always explain in words what you are calculating
f) Ensure that you leave numeric answers in the format required by the question
g) In binary questions, you are advised to group your binary digits into recognisable units of, say, 3 digits. SPACE OUT BINARY CALCULATIONS SO THAT COLUMNS ARE CLEARLY ALIGNED
h) Ensure that you pay particular attention to words underlined, in CAPITALS or in **bold**. FEW OR NO MARKS will be awarded to any question where these are ignored
i) No computer equipment, books or notes may be used in this examination
j) GRAPH PAPER may be issued as required

1. X, Y and Z represent binary numbers as follows:
   X = 1100 Y = 111000 Z = 10001001
   a) Carry out the following calculations **entirely in binary**:
      i X + Y + Z [2]
      ii X * Y [4]
      iii Y / X [5]
   b) Convert the result of EACH of the above calculations to:
      i Decimal [3]
      ii Hexadecimal [3]
      iii Octal [3]

   Note: you must show your working in both part a) and part b) to gain full marks.

2. a) Convert the following decimal numbers into binary, showing how they would EACH be held in a 16-bit register using two's complement format.
      i +4567 [4]
      ii –2468 [4]
   b) A and B are single **unsigned** byte registers. Sometimes these two registers are used together to create a single 2-byte register. A is the more significant one and is considered to be to the left of B.
      If A holds DECIMAL values of 184 and B holds 59, calculate the value of this 2-byte register AB in:
      i decimal
      ii binary [5]
   c) If address P contains the binary value 100 1101 and Q contains 1 1110, perform the following calculations:
      i P + Q
      ii P – Q
      iii P × Q [7]

continued overleaf

3. a) In an arithmetic series the fourth term is 15. The total of the first five terms is 55. Determine the following:
   i The first term [1]
   ii The common difference [3]
   iii The tenth term [1]

   b) A child has 100 wooden blocks, he builds a wall that has one block at the top, two blocks on the next row down, three on the third row and continues with one extra block on each row down. Use a series formula method to calculate the following:
   i How many rows he can build
   ii How many blocks are left over [5]

   c) In an arithmetic progression the seventh term is 4. The second, fifth and eleventh terms are in geometric progression. Find:
   i the first term
   ii the common difference
   iii the common ratio of the geometric progression [8]

   d) A loan of £2,000 is taken out over a period of 4 years. The interest on the loan is 2%. Write down the calculation which would determine the TOTAL amount to be paid back at the end of this period if invested at:
   i simple interest with the annual interest paid back each year
   ii compound interest [2]

4. a) Timings were recorded to the nearest half-minute of 10 people completing a specific task. The timings were as follows:
   8.5, 9.5, 11.5, 7, 8.5, 9, 6.5, 16.5, 6.5, 6.5
   Showing relevant working, calculate the following:
   i Mean [1]
   ii Median [1]
   iii Range [1]
   iv Mode [1]
   v Standard deviation [6]

   b) Create Venn diagrams for EACH of the following situations. Show your reasoning.
   i 26 school children were asked about their pets. The results showed that 14 have dogs, 10 have cats, and 5 have fish. 4 have dogs and cats, 3 have dogs and fish, and 1 has a cat and fish. If no-one has all three kinds of pets, how many children have none of these pets? [5]
   ii 30 students are asked about which languages they intend to study in the forthcoming year. 16 students say they want to take French, 16 want to take Spanish, and 11 want to take Latin. 5 say they want to take both French and Latin, and of these, 3 wanted to take Spanish as well. 5 want only Latin, and 8 want only Spanish. How many students want French only? [5]

5. a)

$$A = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{vmatrix} \qquad B = \begin{vmatrix} 0 & 2 \\ 4 & 6 \\ 1 & 3 \end{vmatrix} \qquad C = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix}$$

   i Perform matrix multiplication A × B. [3]
   ii Write down one other combination of two of these matrices which can be multiplied and perform the multiplication. [4]

   b) Derive an algorithm to add two square matrices each with 3 rows and 3 columns. You may assume that the matrices are already held in two 2-dimensional arrays P and Q and that the sum is to be placed in array R. [5]

   c) X is the matrix

$$\begin{vmatrix} 2 & 3 \\ -1 & -2 \end{vmatrix}$$

   i Perform X.Y and X.Z where:

$$Y = \begin{vmatrix} -4 \\ 4 \end{vmatrix} \quad \text{and} \quad Z = \begin{vmatrix} 4 \\ -4 \end{vmatrix}$$

   [4]

   ii Y is the point (–4,4) and Z the point (4,–4). Show on a sketch diagram the points Y and Z and also their images Y′ and Z′ under the X transformation.
   Describe a SINGLE physical operation which would map Y to Y′ and Z to Z′. [4]

continued overleaf

6. a) Blocks of data labelled A, B, C and D are resident in memory, each one located immediately after the previous one. The table below shows the locations of the data using hexadecimal addressing.

Routine	Start address	End address	Size of data block
A	4000	57FF	
B			
C	7400		
D	8C00	AFFF	

Copy the table and fill in ALL the missing values. Give answers in hexadecimal. [10]

b) A single-address assembly instruction needs 4 bytes of memory (known as a word). It is composed of three parts – operation code, register address and memory address. This computer has 16 working registers occupying decimal addresses 0 to 15.

i If the number of different locations which can be addressed is 1 megabyte, calculate the number of different operation codes possible. [4]

ii Draw up a table showing the THREE component parts of the instruction and numbering the bits used by EACH. [2]

iii If this word can only deal with 128 different operation codes, determine the HIGHEST addressable location now possible. [4]

7. a) A group of students were asked what type of computer game they preferred. The results are summarized below:

	Game types		
Age	Action	Puzzle	Word
14	68	41	46
15	84	56	70
16	59	74	47

Using these students as the sample, a student from this study is randomly selected.

i What is the probability of selecting a student whose favourite type of game is Puzzle?

ii What is the probability of selecting a 14-year-old student?

iii If the student selected is a 15-year-old student, what is the probability that the student prefers Word games?

iv If the student selected prefers Action games, what is the probability that the student is a 14-year-old?

v If the student selected is a 16-year-old, what is the probability that the student prefers Puzzle games or Word games? [10]

b) 2,000 computer users were asked if they own a desktop computer or a laptop. 500 people own a desktop computer, 1,500 own a laptop and 40 own a desktop computer and a laptop.
Find the probability that a person chosen at random owns a laptop, given that the person also owns a desktop computer. [5]

c) The letter X is printed on 8 cards, the letter Y is printed on another 8 cards and the letter Z is printed on another 8 cards. All the cards are laid face down and a person turns over 2 cards at random. What is the probability of selecting 2 DIFFERENT cards? Explain your answer. [5]

8. a) Distinguish clearly between **binary** and **BCD**. As an illustrative example, convert a decimal number of your choice to both binary and BCD. [6]

b) Explain why BCD might be used rather than binary, you could use an example to clarify your answer. [4]

c) Explain what **ASCII** is and why it is used. [4]

d) Write an algorithm (pseudocode or flowchart) for the following:
A string of 6 characters is entered entirely in upper case letters. The string is then converted into its lower case equivalent, e.g. A becomes a. The routine should work for any set of upper case letters.
Note: upper case A = ASCII 65, lower case a = ASCII 97 [6]